Annubar[®] Primary Element Flow Calculations

ANNUBAR PRIMARY ELEMENT FLOW EQUATIONS

The Annubar primary element flow equations are all derived from the hydraulic equations which are shown in equations 2.9 and 2.10 of the Annubar Flow Handbook (document number 00807-0100-1191, Rev BA). For a detailed example of a derivation of an Annubar primary element equation, see the Rosemount 485 Annubar Flow Test Data Book (document number 00809-0100-1193, Rev CA).

Equation 1: Volume rate of flow - Liquids (Actual Conditions)

$$Q_a = C^I \times \sqrt{h_w}$$
 OR $h_w = \left(\frac{Q_a}{C^I}\right)^2$

where:

$$C^{I} = F_{na} \times K \times D^{2} \times F_{aa} \times \sqrt{\frac{1}{G_{f}}}$$

NOTE:

For description of standard volumetric flow equations, see page 2-15 of the Annubar Flow Handbook (document number 00809-0100-1191, Rev BA).

Equation 2: Mass rate of flow - Liquids

$$W = C^{I} \times \sqrt{h_{w}}$$
 OR $h_{w} = \left(\frac{W}{C^{I}}\right)^{2}$

where:

 $C^{I} = F_{na} \times K \times D^{2} \times F_{aa} \times \sqrt{\rho_{f}}$

Equation 3: Mass rate of flow - Gas and Steam

 $W = C^{I} \times \sqrt{h_{w}}$ OR $h_{w} = \left(\frac{W}{C}\right)^{2}$

where:

$$\mathsf{C}^{\mathsf{I}} = \mathsf{F}_{\mathsf{n}\mathsf{a}} \times \mathsf{K} \times \mathsf{D}^{\mathsf{2}} \times \mathsf{Y}_{\mathsf{a}} \times \mathsf{F}_{\mathsf{a}\mathsf{a}} \times \sqrt{\rho_{\mathsf{f}}}$$

Equation 4: Volume rate of flow - Gas (Standard Conditions)

$$Q_s = C^I \times \sqrt{h_w \times P_f}$$
 OR $h_w = \frac{1}{P_f} \times \left(\frac{Q_s}{C^I}\right)^2$

where:

$$C^{I} = F_{na} \times K \times D^{2} \times Y_{a} \times F_{pb} \times F_{tb} \times F_{tf} \times F_{g} \times F_{pv} \times F_{aa}$$

Equation 5: Volume rate of flow - Gas (Actual Conditions)

$$\textbf{Q}_{a} \ = \ \textbf{C}^{I} \times \sqrt{h_{w}} \qquad \textbf{OR} \qquad \qquad \textbf{h}_{w} \ = \ \left(\frac{\textbf{Q}_{a}}{\textbf{C}^{I}} \right)^{2}$$

where:

$$C^{I} = F_{na} \times K \times D^{2} \times Y_{a} \times F_{aa} \times \sqrt{\frac{1}{\rho_{f}}}$$





For a detailed description of each term in the above equations, see "Nomenclature" on page 6. Please note that each of the above equations has a C' constant. It is not intended that the C' constant of one equation is equal to the C' constant of another equation. The numerical value of any C' constant is the product of the appropriate factors for that equation only.

The following tabulations of the flow equations will serve as handy work pads. Also, the table numbers where the necessary information can be found are given in the headings of these tabulations. Several completed examples of flow calculations are given beginning on page 16.

NOTE

The 485 Annubar primary needs no correction for the Reynolds Number.

Rate of Flow	Unit Conversion Factor	Annubar Flow Coefficient	Internal Pipe Diameter	Thermal Expansion Factor (Table 9)	Flowing Specific Gravity	Differential Pressure
		A	nnubar Flow Co	nstant C ^I		
a a	= F _{na} >	×	, D ²	K F _{aa}	x	< \u03cm/h_w
GPM	5.6664		(in) ²		lbm/ft ³	inch H ₂ O at 68 °F
GPH	339.99		(in) ²		lbm/ft ³	inch H ₂ O at 68 °F
GPD	8159.7		(in) ²		lbm/ft ³	inch H ₂ O at 68 °F
BPH (42 gal)	8.0949		(in) ²		lbm/ft ³	inch H ₂ O at 68 °F
BPD (42 gal)	194.28		(in) ²		lbm/ft ³	inch H ₂ O at 68 °F
ft³/min	0.75749		(in) ²		lbm/ft ³	inch H ₂ O at 68 °F
CFH	45.4494		(in) ²		lbm/ft ³	inch H ₂ O at 68 °F
CFM	0.7575		(in) ²		lbm/ft ³	inch H ₂ O at 68 °F
Imp. GPM	4.7183		(in) ²		lbm/ft ³	inch H ₂ O at 68 °F
LPH	0.3958		(mm) ²		kg/m³	mm H ₂ O at 20 °C
LPM	0.0065966		(mm) ²		kg/m³	mm H ₂ O at 20 °C
LPS	0.00010994		(mm) ²		kg/m³	mm H ₂ O at 20 °C
m³/D	0.0094993		(mm) ²		kg/m³	mm H ₂ O at 20 °C
m³/H	0.000396		(mm) ²		kg/m ³	mm H ₂ O at 20 °C
m³/M	6.5967E-06		(mm) ²		kg/m ³	mm H ₂ O at 20 °C
m³/s	1.0995E-07		(mm) ²		kg/m³	mm H ₂ O at 20 °C

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Table 1. Equation for Liquid – Volume Rate of Flow

	e											
	Pressur			at 68 °F	at 68 °F	at 68 °F	at 68 °F	at 20 °C				
	ferential		√hw	ich H ₂ O	1m H ₂ O							
	Dif		×	.1	.1	.=	.1	Ц	Ц	u	u	L
	ensity			~	~	~	~					
	ving De		\overline{P}_{f}	lbm/ft ³	lbm/ft ³	lbm/ft ³	lbm/ft ³	kg/m ³	kg/m³	kg/m³	kg/m ³	kg/m ³
	Flov		×									
	ansion le 9)											
	al Expa or (Tab	7.	Faa									
	Therm Fact	stant C										
	ipe er	w Con	×									
	ernal P Jiamete	oar Flo	D^2	(in) ²	(in) ²	(in) ²	(in) ²	(mm) ²	(mm) ²	(mm) ²	(mm) ²	(mm) ²
5	v Int	Annul	×							-		
	ar Flov ficient		¥									
	Annuk Coef											
5	rsion.		×	S				05	2	-	85	-06
	Conve Factor		F na	8614.5	358.94	5.9823	0.0997	.2511E-	0.3002	0.01251	.00020	4751E-
-4444	, Unit		п					-			0	ω.
i	of Flow		N	Dd	Hdo	ΡM	Sdo	net)/hr	g/D	H/g	g/M	g/S
222	Rate			ι.	Ľ.	д.	Ľ.	T(n	¥	У	x	¥

Equation for Liquid – Mass Rate of Flow Table 2. Table 3. Equation for Gas and Steam- Mass Rate of Flow

Differential Pressure		×	inch H ₂ O at 68 °F	mm H ₂ O at 20 °C							
Flowing Density		< √P _f	lbm/ft ³	lbm/ft ³	lbm/ft ³	lbm/ft ³	kg/m ³				
Thermal Expansion Factor (Table 9)		x F _{aa} >									
Annubar Expansion Factor	w Constant C ^I	< Y _a									
Internal Pipe Diameter	Annubar Flo	× D ² >	(in) ²	(in) ²	(in) ²	(in) ²	(mm) ²	(mm) ²	(mm) ²	(mm) ²	(mm) ²
Annubar Flow Coefficient		×									
Unit Conversion Factor		= F _{na} x	8614.56	358.94	5.9823	0.0997	1.2511E-05	0.30025	0.012511	0.0002085	3.4751E-06
Rate of Flow		~	DD	Hdd	РРМ	PPS	T(met)/hr	kg/D	Kg/H	kg/M	kg/S

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sure			÷	ŕ	÷	S S	S.	с С	с,	с С	с С
Differential Pres		_h_w	inch H ₂ O at 68	inch H ₂ O at 68	inch H ₂ O at 68	mm H ₂ O at 20 '	mm H ₂ O at 20 ⁶				
Flowing Pressure		√P_f x	psia	psia	psia	kPa	kPa	kPa	kPa	kPa	kPa
Thermal Expansion Factor (Table 12)		F _{aa} x									
Supercomp Factor (Table 8)		F _{pv} x									
Specific Gravity Factor		F _g ×									
Flowing Temperature Factor	t C'	F _{tf} x									
Temperature Base Factor	oar Flow Constant	F _{tb} ×									
Pressure Base Factor	Annul	F _{pb} x									
Annubar Expansion Factor		≺ a ×									
Internal Pipe Diameter		D ² x	(in) ²	(in) ²	(in) ²	(mm) ²	mm) ²	(mm) ²	mm) ²	(mm) ²	mm) ²
Annubar Flow Coefficient		× ¥									
Unit Conversion Factor		F _{na} x	8.116.1	338.17	5.6362	1.1227	0.018711	0.026945	0.0011227	1.8712E-05	3.1186E-07
Rate of Flow		° S	SCFD	SCFH	SCFM	NL/H	NL/M	NM ³ /D	H/ ₈ MN	NM ³ /M	NM ³ /S

Table 4. Equation for Gas and Steam- Mass Rate of Flow (Standard Conditions)

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	Differential Pressure		xh	inch H ₂ O at 68 °F	inch H ₂ O at 68 °F	inch H ₂ O at 68 °F	mm H₂O at 20 °C	mm H ₂ O at 20 °C	mm H ₂ O at 20 °C	mm H ₂ O at 20 °C	mm H ₂ O at 20 °C	mm H ₂ O at 20 °C
	Flowing Density		P P	lbm/ft ³	lbm/ft ³	lbm/ft ³	kg/m ³	kg/m ³	kg/m³	kg/m ³	kg/m³	kg/m³
onditions)	Thermal Expansion Factor (Table 9)		F _{aa} x									
ow (Actual C	Annubar Expansion Factor	· Constant C ^I	κ Y _a x									
- Kate of Fi	Internal Pipe Diameter	Annubar Flow	x D ²	(in) ²	(in) ²	(in) ²	(mm) ²	(mm) ²	(mm) ²	(mm) ²	(mm) ²	(mm) ²
s and steam	Annubar Flow Coefficient		×									
equation for Ga	Unit Conversion Factor		= F na	8614.56	358.94	5.9823	12.5100	0.2085	0.30025	0.012511	0.0002085	3.4751E-06
lable 5. E	Rate of Flow		Qa	ACFD	ACFH	ADFM	AL/H	AL/M	Am ³ /D	H/ ₈ mA	W/ ₈ mA	Am ³ /S

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NOMENCLATURE

- D Internal diameter of pipe, inches (mm)
- F_{aa} Thermal Expansion Factor. This factor corrects for the flowing area change of the pipe at the Annubar location due to temperature effects. For 316 stainless steel Annubar primary elements mounted in carbon steel pipe, F_{aa} = 1.0000 for temperatures between 31 and 106 °F. See Table 12 on page 15.
- F_g Specific Gravity Factor. This factor corrects the flow equation whenever the gas is not air. The factor can be calculated as:

 $F_g = \sqrt{\frac{1}{G}}$

where, G = specific gravity of flowing gas, air = 1.000. For a more complete description of specific gravity, see page 2-4 and Appendix C of the Annubar Flow Handbook (document number 00807-0100-1191, Rev BA).

- F_{na} Units Conversion Factor. This factor is used to convert the flow rate to the desired or wanted set of units. Appendix C Annubar Flow Handbook (document number 00807-0100-1191, Rev BA) shows an example of how the numerical value of F_{na} is derived from the hydraulic equation for a given set of input units.
- F_{pb} Pressure Base Factor. The Pressure Base Factors are calculated to give gas volumes at a pressure base of 14.73 psia (101.56 kPa abs). The pressure base factor can be calculated as:

$$F_{pb} = \frac{14.73}{base \text{ pressure, psia}}$$
 OR $F_{pb} = \frac{101.56}{base \text{ pressure, kPa abs}}$

F_{pv} Supercompressibility Factor. The Supercompressibility Factor accounts for the deviation from the "ideal gas" laws. In the flow equations, gas volumes are assumed to vary with pressure and temperature in accordance with Boyle's and Charles' laws (the "ideal gas" laws). Actually, the volume occupied by individual gases deviate, by a slight degree, from the volumes which the "ideal gas" laws indicate. The amount of deviation is a function of the composition of the gas and varies primarily with static pressure and temperature. The actual deviation may be obtained by a laboratory test conducted on a sample of the gas, carefully taken at line conditions of pressure and temperature.

The National Bureau of Standards, Circular 564, gives the compressibility factor (Z) of air and other pure gases. The relationship between supercompressibility factor and compressibility factor is as follows:

$$F_{pv} = \sqrt{\frac{1}{Z}}$$

Table 10 on page 13 gives an abbreviated listing of the supercompressibility factors for air.

Practical relationships have been established by which this deviation can be calculated and tabulated for natural gases containing normal mixtures of hydrocarbon components, considering the presence of small quantities of carbon dioxide and nitrogen and also relating the deviation to the heating value of gas.

The A.G.A. manual (NX-19), "Determination of Supercompressibility Factors for Natural Gas", should be used for determination of F_{pv} .

Table 11 on page 14 gives an abbreviated listing of the supercompressibility factors for natural gas.

F_{tb} Temperature Base Factor. The Temperature Base Factors are calculated to give gas volumes at a base temperature of 60 °F (520°R) or 16 °C (289 K). The factor can be calculated as:

 $F_{tb} = \frac{temperature base (°F) + 460}{520}$ OR $F_{tb} = \frac{temperature base (°C) + 273}{289}$

F_{tf} Flowing Temperature Factor. The units conversion factor (F_{na}) for volumetric flow of gases at standard conditions has been calculated assuming that the gas temperature flowing around the Annubar primary element is 60 °F (520°R) or 16 °C (289 K). If measurement is made at any other flowing temperature, then the flowing temperature factor must be applied. The factor can be calculated as:

$$F_{tf} = \sqrt{\frac{520}{\text{flowing temperature (°F) + 460}}} \qquad \text{OR} \qquad F_{tf} = \sqrt{\frac{289}{\text{flowing temperature (°C) + 273}}}$$

- G Specific Gravity of Flowing Liquid. Ratio of the density of the flowing fluid to the density of water at 60°F which is 63.3707 lbm/ft³. See Appendix A of the Annubar Flow Handbook (document number 00807-0100-1191, Rev BA) for specific gravities of various liquids.
- h_w Differential pressure as measured by the Annubar primary element. For this handbook, the differential pressure is expressed as the height, in inches, of a water column at 68 °F at standard gravity ($g_c = 32.174$ ft/sec² = 9.807 m/sec²).

 h_w = inches H₂O at 68 °F (mm at 20 °C)

- K Flow Coefficient. Equation 8 on page 10 defines the flow coefficient of an Annubar primary element. It is related to the diameter of the pipe and is generally expressed as a function of Reynolds Number. See page 2-7 of the Annubar Flow Handbook (document number 00807-0100-1191, Rev BA) for an explanation of Reynolds Number.
- P_f Flowing Pressure. This is the static pressure, in absolute units, existing in the pipe. For this handbook, the pressures are expressed in psia and kPa abs.
- Q_a Actual Volumetric Flow Rate. This term is the flow rate of the fluid passing the Annubar primary element in actual volume units per units of time. Examples are actual cubic feet per hour (ACFH), GPM, etc.
- Q_s Standard Volumetric Flow Rate. This term is the flow rate of the fluid passing the Annubar primary element in standard volume units per unit of time. For some gases, especially fuel gases, the cubic foot is the unit of measurement. A cubic foot of gas has no absolute or comparative value unless the pressure and temperature of the gas are specified when it fills a cubic foot. A common unit used for evaluating rates of flow is "standard cubic foot per hour," (SCFH). This unit states how many cubic feet of gas per hour would be flowing around the Annubar primary element if the flowing pressure and temperature were equal to the base pressure and temperature. For this handbook, the base pressure is 14.73 psia (101.56 kPa abs) and the base temperature is 60 °F (520°R) or 16°C (289 K).

- W Mass Rate of Flow. This term is the flow rate of the fluid passing the Annubar primary element in mass units per unit time.
- Y_A Expansion Factor. When a gas flows around an Annubar primary element, the change in velocity is accompanied by a change in density. The expansion factor must be applied to correct for this change. The expansion factor also accounts for small changes in the internal energy of the molecules due to the temperature difference between the upstream and downstream pressure ports of the Annubar primary element. The variation of the expansion factor is small and the ratio of specific heats for commercial gases is sufficiently constant to warrant using a constant ratio of specific heat. Use the following algorithm to calculate the value of the gas expansion factor. This equation adjusts for density and internal energy effects of the gas as it flows around the Annubar primary element.

Equation 6: Gas Expansion Factor

$$Y_{a} = 1 - (0.011332(1 - B)^{2} - 0.00342) \frac{n_{w}}{P_{f} \Upsilon}$$

where:

Equation 7: Blockage Equation

 $\begin{array}{l} \mathsf{B} = \frac{4\mathsf{d}}{\pi\mathsf{D}} = \mathsf{Blockage} \\ \mathsf{D} = \mathsf{Internal} \; \mathsf{Pipe} \; \mathsf{Diameter} \; \mathsf{in} \; \mathsf{inches} \; (\mathsf{cm}) \\ \mathsf{d} = \mathsf{See} \; \mathsf{Table} \; \mathsf{Table} \; \mathsf{8} \; \mathsf{on} \; \mathsf{page} \; \mathsf{12} \\ \mathsf{h}_{\mathsf{W}} = \mathsf{Differential} \; \mathsf{pressure} \; \mathsf{in} \; \mathsf{inches} \; (\mathsf{mm}) \; \mathsf{of} \; \mathsf{water} \; \mathsf{column} \\ \mathsf{P}_{\mathsf{F}} = \mathsf{Flowing} \; \mathsf{line} \; \mathsf{pressure} \; \mathsf{in} \; \mathsf{psia} \; (\mathsf{kPa} \; \mathsf{abs}) \\ \gamma = \mathsf{Ratio} \; \mathsf{of} \; \mathsf{specific} \; \mathsf{heats} \end{array}$

Examples of gases with a specific heat ratio of 1.4 are: air, CO, H_2 , NO, N_2 and O_2 . Examples of gases with a specific heat ratio of 1.3 are: natural gas, ammonia, CO₂, Cl₂, H₂S, N₂O, SO₂, and steam.

 Y_a is needed in all gas flow equations and requires the differential pressure can be calculated first. If the differential pressure is not known, Y_a is assumed to be 1.000 and the differential pressure is calculated. Iteration is then necessary to determine its final value.

ρ_f Flowing Density. For this handbook, the densities are expressed in lbm/ft (kg/m³). Appendix A of the Annubar Flow Handbook (document number 00807-0100-1191, Rev BA gives densities of various fluids.

Flow Coefficient Reynolds Number Dependency	When the Rosemount 485 Annubar primary element is used within the acceptable Reynolds Number range defined by Rosemount, the Annubar Primary element's flow coefficient will be independent of changing Reynolds Number. Any variations in the K-value with changing Reynolds Number are due to scatter and fall within ±.75% of the published K-value.
	A 485 Annubar primary element's K-factor independence of Reynolds number allows the user to measure a large range of Reynolds Numbers without need of a correction factor. The 485 Annubar's K-factor independence can be attributed to a constant separation point along the edges of its T-shaped sensor and the probe's ability to take a proper average of its sensing slots.
Flow Coefficient Theory	Rosemount is the first company to identify and utilize the theoretical equations linking self-averaging pitot tube flow coefficients to pipe blockage. This K-to-Blockage theoretical link establishes a higher degree of confidence in 485 Annubar K-factors than in flow meters that use only an empirical data base for determining their flow coefficients.
Signal	The signal generated by an Annubar primary element can be divided into two major parts:
	 the differential pressure contribution due to the Annubar primary element's shape (H_S)
	 the differential pressure contribution due to the Annubar primary element's blockage in the pipe (H_b).
How Sensor Shape Affects the Differential Pressure	An Annubar primary element placed in an infinitely large pipe (with no confining walls) will still produce a differential pressure. This differential pressure is nearly twice that of a standard pitot tube, and is the result of a reduced low pressure on the downstream side. The upstream, or high pressure is caused by the fluid impacting the front of the Annubar primary element and is known as the stagnation pressure. The downstream, or low pressure is caused by the fluid traveling past the Annubar primary element, creating suction on the rear side. This suction phenomenon can be attributed to a boundary layer flow separation.
How Pipe Blockage Affects Differential Pressure	An Annubar primary element is an obstruction in the pipe and therefore, reduces the cross-sectional area in which the fluid can pass. This reduced area causes the fluid to accelerate and hence, reduces its pressure. Therefore, the downstream pressure measurement of an Annubar primary element will be affected by the Annubar primary element's blockage in the pipe.
	Since an Annubar primary element uses the internal diameter of the pipe it is being inserted into as a throat diameter in its calculation of a flow rate, the Annubar primary element K-factor must compensate for the amount of obstructed area the sensor itself causes in the pipe. This is analogous to the velocity of approach factor for an orifice plate or a venturi meter.
	By writing a mass balance and an energy balance around the Annubar primary element, and by dividing the differential pressure produced by the Annubar primary element into H_s and H_b , one can derive the relationship between the Annubar primary element K-factor and the Annubar primary element's blockage in the pipe. The derivation involves partial differential pressure components, and the integration of a k-blockage equation. The result is the following K vs. Blockage equation:

Equation 8: K vs. Blockage

$$K_{A} = \frac{(1 - C_{2}B)}{\sqrt{1 - C_{1}(1 - C_{2}B)^{2}}}$$

The constants C_1 and C_2 must be determined experimentally. Once C_1 and C_2 are determined, the equations above becomes the theoretical link between the Annubar primary element K-factor (K) and the Annubar primary element's blockage in the pipe (B). The values for constants C_1 and C_2 are shown in the table below:

Table 6. 485 Sensor Constants

Coefficient	Sensor Size 1	Sensor Size 2	Sensor Size 3
C ₁	- 1.515	- 1.492	- 1.5856
C ₂	1.4229	1.4179	1.3318

The Importance of the Flow Coefficient, or K vs. Blockage Theory

As with any other meter, the 485 Annubar primary element's accuracy is only as good as its flow coefficient (K-factor). Rosemount has tested a representative sample of Flowmeters and empirically determined flow coefficients. For Annubars, these flow coefficients are plotted against the meter's blockage. Curve fitting techniques are applied to the base line data to generate equations that predict flow coefficients in untested line sizes and untested Reynolds Number ranges. Please see the 485 Annubar Flow Test Data Book (document number 00809-0100-1193, Rev CA) for a more detailed discussion of this topic.

Provided the theory is based on the proper physics, these relationships are immune to minor variation in test data. Using a theoretical basis (in addition to empirical testing) for the prediction of untested flow coefficients provides a much higher degree of confidence in the untested values. The following graphs show that empirical data agree with a plot of the K vs. Blockage Equation.

Figure 7. K vs. BLOCKAGE



Sensor Size 2 K vs. Blockage



Sensor Size 3 K vs. Blockage



Operating Limitations

For an Annubar primary element to operate accurately, the flowing fluid must be separate from the probe at the same location (along the edges of the T-shape sensor). Drag coefficients, lift coefficients, separation points, and pressure distributions around bluff bodies are best compared by calculating the "rod" Reynolds Number. There is a minimum rod Reynolds Number at which the flowing fluid will not properly separate from the edges of a T-shape sensor. The minimum rod Reynolds Numbers for the Rosemount 485 are:

Table 8. Reynolds Number and Probe Width

Sensor Size	Probe Width (d)	Minimum Reynolds Number
1	0.590-in. (1.4986 cm)	6500
2	1.060-in (2.6924 cm)	12500
3	1.920-in (4.8768 cm)	25000

Above these rod Reynolds Numbers, 485 Annubar primary elements will operate accurately. To determine the rod Reynolds Number at any given flowrate, use the following relationship:

$$\operatorname{Re}_{\operatorname{rod}} = \frac{\mathrm{d}V\rho}{12\mu}$$
 OR $\operatorname{Re}_{\operatorname{rod}} = \frac{\mathrm{d}V\rho}{100\mu}$

where,

 ρ = fluid density in lbm/ft³ (kg/m³)

d = probe width in inches (cm)

V = velocity of fluid in feet per second (m/s)

 μ = fluid viscosity in lbm/ft-sec (kg/m-s)

When determining the minimum operating flow rate for an Annubar primary element, one should also consider the capability of the secondary instrumentation (differential pressure transmitters, manometers, etc.).

The upper operating limit for Rosemount 485 Annubar primary elements is reached when any one of the following criteria is met:

- 1. The fluid velocity reaches the structural limit of the Annubar primary element.
- 2. The fluid velocity reaches a choked flow condition at the Annubar primary element (for gases).
- 3. Cavitation occurs on the downstream side of the Annubar primary element.

Table 9. Supercompressibility Factor, F_{pv}

	Range of Specific	Cons	stants
	Gravity, G	k1	k2
$5 + k_2G$	0.600 < G <	2.480	2.020
$F = 1 + \frac{k_1 \rho_G (10^2)}{k_1 \rho_G (10^2)}$	0.601 < G < 0.650	3.320	1.810
^{pv} √ T _f ^{3.825}	0.651 < G < 0.750	4.660	1.600
,	0.751 < G < 0.900	7.910	1.260
	0.901 < G < 1.100	11.630	1.070
	1.101 < G < 1.500	17.480	0.900

Tabla 10	Supercompressibilit	V Eactor Air	Spacific	Cravity 1.0
	Supercompression	y i actoi. All,	Specific	Gravity 1.0

Pressure							Tempe	rature						
psia	–40 °F	–20 °F	0 °F	20 °F	40 °F	60 °F	80 °F	100 °F	120 °F	140 °F	160 °F	180 °F	200 °F	220 °F
14.7	1.0092	1.0077	1.0065	1.0056	1.0048	1.0041	1.0035	1.0031	1.0027	1.0024	1.0021	1.0019	1.0016	1.0015
100	1.0613	1.0516	1.0437	1.0372	1.0319	1.0275	1.0239	1.0208	1.0182	1.0160	1.0141	1.0125	1.0112	1.0100
200	1.1193	1.1007	1.0856	1.0732	1.0629	1.0544	1.0472	1.0412	1.0361	1.0318	1.0281	1.0249	1.0222	1.0198
300	1.1744	1.1477	1.1259	1.1079	1.0930	1.0805	1.0700	1.0612	1.0537	1.0473	1.0419	1.0372	1.0331	1.0296
400	1.2270	1.1929	1.1649	1.1416	1.1223	1.1060	1.0924	1.0808	1.0710	1.0626	1.0554	1.0493	1.0439	1.0392
500	1.2774	1.2365	1.2026	1.1744	1.1508	1.1310	1.1143	1.1001	1.0880	1.0777	1.0689	1.0612	1.0546	1.0488
600	1.3260	1.2785	1.2391	1.2062	1.1786	1.1554	1.1358	1.1191	1.1048	1.0926	1.0821	1.0730	1.0652	1.0583
700	1.3728	1.3192	1.2746	1.2373	1.2058	1.1793	1.1568	1.1377	1.1213	1.1073	1.0952	1.0847	1.0756	1.0677
800	1.4181	1.3587	1.3091	1.2675	1.2324	1.2028	1.1775	1.1560	1.1376	1.1218	1.1081	1.0963	1.0860	1.0771
900	1.4620	1.3971	1.3428	1.2971	1.2585	1.2258	1.1979	1.1741	1.1537	1.1361	1.1209	1.1077	1.0963	1.0863
1000	1.5046	1.4345	1.3756	1.3260	1.2840	1.2483	1.2179	1.1918	1.1695	1.1502	1.1335	1.1191	1.1065	1.0955
1100	1.5460	1.4709	1.4077	1.3543	1.3090	1.2705	1.2376	1.2094	1.1851	1.1642	1.1460	1.1303	1.1166	1.1046
1200	1.5863	1.5064	1.4390	1.3820	1.3336	1.2923	1.2569	1.2266	1.2005	1.1779	1.1584	1.1414	1.1266	1.1136
1300	1.6257	1.5411	1.4697	1.4092	1.3577	1.3137	1.2760	1.2436	1.2157	1.1916	1.1706	1.1524	1.1365	1.1226
1400	1.6641	1.5751	1.4998	1.4358	1.3814	1.3348	1.2948	1.2604	1.2308	1.2051	1.1827	1.1633	1.1463	1.1314
1470	1.6905	1.5984	1.5204	1.4542	1.3977	1.3493	1.3078	1.2721	1.2412	1.2144	1.1911	1.1709	1.1531	1.1376

Pressure							Tempe	erature						
psia	–40 °F	–20 °F	0 °F	20 °F	40 °F	60 °F	80 °F	100 °F	120 °F	140 °F	160 °F	180 °F	200 °F	220 °F
14.7	1.0027	1.0023	1.0019	1.0016	1.0014	1.0012	1.0011	1.0009	1.008	1.0007	1.0006	1.005	1.0005	1.0004
100	1.0185	1.0155	1.0131	1.0112	1.0096	1.0082	1.0071	1.0062	1.0054	1.0048	1.0042	1.0037	1.0033	1.0030
200	1.0368	1.0309	1.0261	1.0222	1.0190	1.0164	1.0142	1.0124	1.0108	1.0095	1.0084	1.0074	1.0066	1.0059
300	1.0547	1.0459	1.0389	1.0331	1.0284	1.0245	1.0212	1.0185	1.0162	1.0142	1.0126	1.0111	1.0099	1.0089
400	1.0722	1.0608	1.0515	1.0440	1.0377	1.0325	1.0282	1.0246	1.0215	1.0190	1.0167	1.0148	1.0132	1.0118
500	1.0896	1.0755	1.0640	1.0547	1.0469	1.0405	1.0352	1.0307	1.0269	1.0236	1.0209	1.0185	1.0165	1.0147
600	1.1066	1.0899	1.0764	1.0652	1.0561	1.0484	1.0421	1.0367	1.0322	1.0283	1.0250	1.0222	1.0197	1.0176
700	1.1234	1.1042	1.0886	1.0757	1.0651	1.0563	1.0489	1.0427	1.0374	1.0329	1.0291	1.0258	1.0230	1.0205
800	1.1399	1.1183	1.1007	1.0861	1.0741	1.0641	1.0557	1.0486	1.0427	1.0376	1.0332	1.0295	1.0262	1.0234
900	1.1562	1.1322	1.1126	1.0964	1.0830	1.0718	1.0625	1.0546	1.0479	1.0422	1.0373	1.0331	1.0295	1.0263
1000	1.1723	1.1460	1.1244	1.1066	1.0918	1.0795	1.0692	1.0604	1.0530	1.0467	1.0413	1.0367	1.0327	1.0292
1100	1.1882	1.1596	1.1361	1.1167	1.1006	1.0871	1.0758	1.0663	1.0582	1.0513	1.0454	1.0403	1.0359	1.0321
1200	1.2038	1.1730	1.1477	1.1267	1.1093	1.0947	1.0825	1.0721	1.0633	1.0558	1.0494	1.0439	1.0391	1.0349
1300	1.2193	1.1863	1.1591	1.1366	1.1179	1.1023	1.0891	1.0779	1.0684	1.0604	1.0534	1.0474	1.0423	1.0378
1400	1.2345	1.1994	1.1705	1.1465	1.1265	1.1097	1.0956	1.0837	1.0735	1.0649	1.0574	1.0510	1.0455	1.0407
1500	1.2496	1.2124	1.1817	1.1562	1.1350	1.1172	1.1021	1.0894	1.0786	1.0693	1.0614	1.0546	1.0486	1.0435
1600	1.2645	1.2253	1.1928	1.1659	1.1434	1.1245	1.1086	1.0951	1.0836	1.0738	1.0654	1.0581	1.0518	1.0463
1700	1.2792	1.2380	1.2039	1.1755	1.1518	1.1319	1.1150	1.1008	1.0886	1.0782	1.0693	1.0616	1.0550	1.0492
1800	1.2937	1.2506	1.2148	1.1850	1.1601	1.1391	1.1214	1.1064	1.0936	1.0827	1.0733	1.0651	1.0581	1.0520
1900	1.3081	1.2630	1.2256	1.1945	1.1683	1.1464	1.1278	1.1120	1.0986	1.0871	1.0772	1.0686	1.0612	1.0548
2000	1.3223	1.2754	1.2364	1.2038	1.1765	1.1536	1.1341	1.1176	1.1035	1.0915	1.0811	1.0721	1.0644	1.0576
2100	1.3364	1.2876	1.2470	1.2131	1.1847	1.1607	1.1404	1.1232	1.1085	1.0958	1.0850	1.0756	1.0675	1.0604
2200	1.3503	1.2997	1.2576	1.2223	1.1928	1.1678	1.1467	1.1287	1.1134	1.1002	1.0889	1.0791	1.0706	1.0632
2300	1.3641	1.3117	1.2680	1.2315	1.2008	1.1749	1.1529	1.1342	1.1182	1.1045	1.0927	1.0825	1.0737	1.0660
2400	1.3778	1.3236	1.2784	1.2406	1.2088	1.1819	1.1591	1.1397	1.1231	1.1089	1.0966	1.0860	1.0768	1.0688
2500	1.3913	1.3354	1.2887	1.2496	1.2167	1.1889	1.1653	1.1451	1.1279	1.1132	1.1004	1.0894	1.0799	1.0715
2600	1.4047	1.3470	1.2989	1.2585	1.2245	1.1958	1.1714	1.1506	1.1328	1.1175	1.1043	1.0928	1.0829	1.0743
2700	1.4180	1.3586	1.3090	1.2674	1.2324	1.2027	1.1775	1.1560	1.1376	1.1217	1.1081	1.0963	1.0860	1.0770
2800	1.4311	1.3701	1.3191	1.2763	1.2401	1.2095	1.1835	1.1613	1.1423	1.1260	1.1119	1.0997	1.0890	1.0798
2900	1.4441	1.3815	1.3291	1.2850	1.2479	1.2164	1.1896	1.1667	1.1471	1.1302	1.1157	1.1031	1.0921	1.0825
3000	1.4570	1.3928	1.3390	1.2937	1.2555	1.2231	1.1956	1.1720	1.1518	1.1344	1.1194	1.1064	1.0951	1.0853

Table 11. Supercompressibility Factor: Hydrocarbon Gas, Specific Gravity 0.6

Table 12. F_{aa} Thermal Expansion Factor

Temperature (°F) of Piping Material									
Aluminum	Copper	Type 430	2% CRMO	5% CRMO	Bronze	Carbon Steel	Туре 316 Туре 304	Correcton Factor, F _{aa}	
- 264					- 317			0.993	
- 204	- 322				- 245			0.994	
– 155	- 230				- 190		- 276	0.995	
- 108	- 163				– 137		– 189	0.996	
- 63	- 102				- 86		– 119	0.997	
– 19	- 44				- 34		- 55	0.998	
25	19	44	– 13	- 14	17	- 6	7	0.999	
68	68	68	68	68	68	68	68	1.000	
113	127	157	146	151	122	144	130	1.001	
		246	222	232	175	218	186	1.002	
		332	296	312	225	289	240	1.003	
		415	366	389	273	358	292	1.004	
		494	434	460	321	425	343	1.005	
		568	501	527	369	489	391	1.006	
		641	566	594	417	551	439	1.007	
		713	629	662		613	488	1.008	
		783	690	730		675	536	1.009	
		851	750	795		735	584	1.010	
		918	811	858		794	631	1.011	
		956	871	918		851	674	1.012	
		1054	928	979		907	727	1.013	
		1121	984	1040		961	777	1.014	
		1189	1038	1102		1015	799	1.015	

Flow Calculation Examples:

Problem:

Oil with a specific gravity of 0.825 is flowing at a rate of 6000 GPM. The 20-in. standard wall (ID - 19.26-in.) carbon steel pipeline has a pressure of 75 psig and a temperature of 100°F. What is the differential pressure (h_w) that a Sensor Size 2 Rosemount 485 Annubar primary element would measure?

Solution:

 $h_w \, = \, \left(\frac{Q_a}{C^1} \right)^2$

(from Equation 1 on page 1)

Q_a = 600 GPM

$$C^{I} = F_{na} \times K \times D^{2} \times F_{aa} \times \sqrt{\frac{1}{G_{aa}}}$$

 $F_{na} = 5.6664$

where:

(from Table 1 on page 3)

$$\kappa = \frac{(1 - C_2 B)}{\sqrt{1 - C_1 (1 - C_2 B)^2}}$$
 (from Equation 8 on page 10)

where:

$$B = \frac{4d}{\pi D} = \frac{4(1.060)}{19.25\pi} = 0.0701$$
 (from Equation 7 on page 8)

(from Table 6 on page 10)

$$C_2 = 1.4179$$

 $C_1 = -1.492$

SO:

$$K = \frac{(1 - 1.4179 \times 0.0701)}{\sqrt{1 - (-1.492) \times (1 - 1.4179 \times 0.0701)^2}} = 0.6058$$
$$D^2 = 19.26^2 = 370.9476$$
$$F_{aa} = 1.000 \qquad (from Table 6 on page 10)$$

 $\sqrt{\frac{1}{G_{\rm f}}} = \sqrt{\frac{1}{0.825}} = 1.101$

SO:

 $C^{I} = 5.6664 \times 0.6058 \times 370.9476 \times 1.000 \times 1.101 = 1401.9625$

and:

$$h_w = \left(\frac{6000}{1401.9625}\right)^2 = 18.316$$
 inchH₂O

Oil with a specific gravity of 0.825 is flowing at a rate of 22,700 LPM. The 50 cm inside diameter carbon steel pipeline has a pressure of 517 kPa and a temperature of 38 °C. What is the differential pressure (h_w) that a Sensor Size 2 Rosemount 485 Annubar primary element would measure?

Solution:

 $h_w = \left(\frac{Q_a}{C^1}\right)^2$

(from Equation 1 on page 1)

Q_a = 22700 LPM

 $C^{I} = F_{na} \times K \times D^{2} \times F_{aa} \times \sqrt{\frac{1}{G_{f}}}$

where:

$$F_{na} = 0.0065966 (from Table 1 on page 3)$$

$$K = \frac{(1 - C_2 B)}{\sqrt{1 - C_1 (1 - C_2 B)^2}} (from Equation 8 on page 10)$$

where:

$$B = \frac{4d}{\pi D} = \frac{4(2.6924)}{50\pi} = 0.0686 \text{ (from Equation 7 on page 8)}$$

$$C_1 = -1.492 \text{ (from Table 6 on page 10)}$$

$$C_2 = 1.4179$$

SO:

$$\begin{split} &\mathsf{K} = \frac{(1-1.4179 \times 0.0686)}{\sqrt{1-(-1.492) \times (1-1.4179 \times 0.0686)^2}} = 0.6065 \\ &\mathsf{D}^2 = 500^2 = 250000 \\ &\mathsf{F}_{aa} = 1.000 \qquad \textit{(from Table 6 on page 10)} \\ &\sqrt{\frac{1}{\mathsf{G}_f}} = \sqrt{\frac{1}{0.825}} = 1.101 \end{split}$$

SO:

 $C^{I} \ = \ 0.0065966 \times 0.6065 \times 250000 \times 1.000 \times 1.101 \ = \ 1101.23$

and:

$$h_w = \left(\frac{22700}{1101.23}\right)^2 = 424.91 mmH_2O$$

Steam at 500 psia and 620 °F is flowing in a 24-in. ID carbon steel pipe. The measured differential pressure on a Sensor Size 3 Rosemount 485 Annubar primary element is 15-in H_2O . What is the flowrate in PPH?

Solution:

$$= C^{I} \times \sqrt{h_{w}}$$
 (from Equation 3 on page 1)
= $F_{na} \times K \times D^{2} \times Y_{a} \times F_{aa} \times \sqrt{\rho_{f}}$

where:

W CI

 $F_{na} = 358.94 \qquad (from Table 1 on page 3)$ $K = \frac{(1 - C_2 B)}{\sqrt{1 - C_1 (1 - C_2 B)^2}} \qquad (from Equation 8 on page 10)$

where:

$$B = \frac{4d}{\pi D} = \frac{4(1.920)}{24\pi} = 0.1019$$
 (from Equation 7 on page 8)

$$C_2 = 1.3318$$

 $C_1 = -1.5856$

SO:

$$K = \frac{(1 - 1.3318 \times 0.1019)}{\sqrt{1 - (-1.5856) \times (1 - 1.3318 \times 0.1019)^2}} = 0.5848$$

$$D^{2} = 24^{2} = 576$$

 $Y_{a} = 1 - (0.011332(1 - B)^{2} - 0.00342) \frac{h_{w}}{P_{f}\Upsilon}$ (from Equation 6 on page 8)

where:

$$B = \frac{4d}{\pi D} = \frac{4(1.920)}{24\pi} = 0.1019$$
$$H_{w} = 15inH_{2}O$$
$$P_{f} = 500psia$$
$$\Upsilon = 1.3$$

SO:

$$Y_{a} = 1 - (0.011332(1 - 0.1019)^{2} - 0.00342) \frac{15}{500 \times 1.3} = 0.9999$$

$$F_{aa} = 1.008$$

$$\sqrt{\rho_{f}} = \sqrt{0.8413} = 0.9172$$

so

 $\textbf{C}^{I} = \ \textbf{358.94} \times \textbf{0.5848} \times \textbf{576} \times \textbf{0.9999} \times \textbf{1.008} \times \textbf{0.9172} = \ \textbf{111771.96}$

 $W = 111771.96 \times \sqrt{15} = 432890.93$ PPH

Steam at 3500 kPa abs and 350 °C is flowing in a 60.96 cm ID carbon steel pipe. The measured differential pressure on a Sensor Size 3 Rosemount 485 Annubar primary element is 715.04 mm H_2O . What is the flowrate in kg/hr?

Solution:)

where:

$$F_{na} = 0.012511 \qquad (from Table 3 on page 4)$$

$$K = \frac{(1 - C_2 B)}{\sqrt{1 - C_1 (1 - C_2 B)^2}} \qquad (from Equation 8 on page 10)$$

where:

$$B = \frac{4d}{\pi D} = \frac{4(4.9149)}{60.96\pi} = 0.1027$$
 (from Equation 7 on page 8)

$$C_2 = 1.3318$$

 $C_1 = -1.5856$

so:

$$K = \frac{(1 - 1.3318 \times 0.1027)}{\sqrt{1 - (-1.5856) \times (1 - 1.3318 \times 0.1027)^2}} = 0.5848$$

$$D^{2} = 609.6^{2} = 371612.16$$

 $Y_{a} = 1 - (0.011332(1 - B)^{2} - 0.00342) \frac{h_{w}}{P_{f} \Gamma}$ (from Equation 6 on page 8)

where:

$$B = \frac{4d}{\pi D} = \frac{4(4.9149)}{60.96\pi} = 0.1027$$
$$H_{w} = 715.04 \text{mm}H_{2}\text{O}$$
$$P_{f} = 3500 \text{psia}$$
$$\Upsilon = 1.3$$

SO:

$$\begin{split} & Y_a = 1 - (0.011332(1 - 0.1027)^2 - 0.00342) \frac{715.04}{3500 \times 1.3} = 0.9991 \\ & F_{aa} = 1.000 \\ & \sqrt{\rho_f} = \sqrt{13.0249} = 3.609 \qquad (P_f = 13.0249 \text{ kg/m}^3 \text{ per ASME Steam Tables}) \end{split}$$

so

 $C^{I} = \ 0.012511 \times 0.5848 \times 371612.16 \times 0.9991 \times 1.000 \times 3.609 \ = \ 9803.59$

$$W = 9803.59 \times \sqrt{715.04} = 262150.27 \qquad (kg)/h$$

Natural gas with a specific gravity of 0.63 is flowing in a 12-in. schedule 80 carbon steel pipe. the operating pressure is 1264 psia and he operating temperature is 120 °F. For a Sensor Size 2 Rosemount 485 Annubar primary element, determine the differential pressure (h_w) for a flowrate of 6 MM SCFH at a base temperature of 60 °F and a pressure of 14.73 psia.

Solution:

$$h_{w} = \frac{1}{P_{f}} \times \left(\frac{Q_{s}}{C^{l}}\right)^{2}$$

(from Equation 4 on page 1)

 $Q_{s} = 600000SCFH$

 $C^{I} = F_{na} \times K \times D^{2} \times Y_{a} \times F_{pb} \times F_{tb} \times F_{tf} \times F_{g} \times F_{pv} \times F_{aa}$

where:

(from Table 4 on page 5)

where:

F_{na} = 338.11

$B = \frac{4d}{\piD} = \frac{4(1.060)}{11.37\pi} = 0.1186$	(from Equation 7 on page 8)
$C_1 = -1.492$	(from Table 6 on page 10)

$$C_2 = 1.4179$$

 $K = \frac{(1 - C_2 B)}{\sqrt{1 - C_1 (1 - C_2 B)^2}}$

SO:

$$K = \frac{(1 - 1.4179 \times 0.1186)}{\sqrt{1 - (-1.492) \times (1 - 1.4179 \times 0.1186)^2}} = 0.5835$$

$$D^2 = 11.376^2 = 129.41$$

The differential pressure h_w is required to calculate Y_a . Since h_w is not known, assume $Y_a = 1$ and verify the results.

$$\begin{aligned} F_{pb} &= \frac{14.73}{base \text{ pressure, psia}} &= \frac{14.73}{14.73} = 1 \\ F_{pb} &= \frac{temperature base (°F) + 460}{520} &= \frac{60 + 460}{520} = 1 \\ F_{tf} &= \sqrt{\frac{520}{flowing \text{ temperature (°F) + 460}}} &= \sqrt{\frac{520}{120(520) + 460}} = 0.9469 \\ F_{g} &= \sqrt{\frac{1}{G}} = \sqrt{\frac{1}{0.63}} = 1.2599 \\ F_{pv} &= \sqrt{\frac{1}{Z}} = \sqrt{\frac{1}{0.8838}} = 1.0637 \end{aligned}$$

$$\begin{aligned} Compressibility factor for natural gas from A.G.A \\ Report Number 8 \\ F_{aa} &= 1.001 \end{aligned}$$

$$(from Table 12 \text{ on page 15})$$

SO:

$$C^{I} = 338.17 \times 1.5835 \times 129.41 \times 1 \times 1 \times 1 \times 0.9469 \times 1.2599 \times 1.0637 \times 1.001 = 32436.74$$
$$h_{w} = \frac{1}{P_{f}} \times \left(\frac{Q_{s}}{C^{I}}\right)^{2} = \frac{1}{1264} \times \left(\frac{6000000}{32436.74}\right)^{2} = 27.07 \text{in}H_{2}O$$

Now the value of Y_a, assumed above, can be checked:

 $Y_a = 1 - (0.011332(1 - B)^2 - 0.00342) \frac{h_w}{P_f \Upsilon}$ (from Equation 6 on page 8)

where

 $B = \frac{4d}{\pi D} = \frac{4(1.060)}{11.37\pi} = 0.1186$ $H_{w} = 27.07 \text{inch}H_{2}O$ $P_{f} = 1264 \text{psia}$ $\Upsilon = 1.3$

SO:

 $Y_{a} = 1 - (0.011332(1 - 1186)^{2} - 0.00342)\frac{27.07}{1264 \times 1.3} = 1$

The assumed and calculated value are the same. Therefore, the value of h_w = 27.07 inch H₂O is the correct answer.

Natural gas with a specific gravity of 0.63 is flowing in a 330 mm ID carbon steel pipe. The operating pressure is 8700 kPA abs and he operating temperature is 50 °C. For a Sensor Size 2 Rosemount 485 Annubar primary element, determine the differential pressure (h_w) for a flowrate of 1700 Nm³/m at a base temperature of 16 °C and a pressure of 103 kPa.

Solution:

$$Q_s = 1700((Nm^3)/m)$$

F_{na} = 1.8712 x 10⁻⁵

$$P_f = 8700 k Pa$$

 $h_w = \frac{1}{P_f} \times \left(\frac{Q_s}{C^l}\right)^2$

 $C^{I} = F_{na} \times K \times D^{2} \times Y_{a} \times F_{pb} \times F_{tb} \times F_{tf} \times F_{a} \times F_{pv} \times F_{aa}$

where:

$$K = \frac{(1 - C_2 B)}{\sqrt{1 - C_1 (1 - C_2 B)^2}}$$
 (from Equation 8 on page 10)

where:

$$B = \frac{4d}{\pi D} = \frac{4(26.924)}{300\pi} = 0.1143$$
 (from Equation 7 on page 8)

$$C_1 = -1.492$$

$$C_2 = 1.4179$$
 (from Table 6 on page 10)

so:

$$K \; = \; \frac{1 - 1.4179 \times 0.1143}{\sqrt{1 - (-1.492) \times (1 - 1.4179 \times 0.1143)^2}} \; = \; 0.5856$$

$$D^2 = 11.376^2 = 129.41$$

The differential pressure h_w is required to calculate Y_a . Since h_w is not known, assume $Y_a = 1$ and verify the results.

$$\begin{aligned} F_{pb} &= \frac{101.56}{\text{base pressure, kPa abs}} = \frac{101.56}{103} = 0.9860 \\ F_{pb} &= \frac{\text{temperature base (°C) + 273}}{289} = \frac{16 + 273}{289} = 1 \\ F_{tf} &= \sqrt{\frac{289}{\text{flowing temperature (°C) + 273}}} = \sqrt{\frac{289}{50 + 273}} = 0.9459 \\ F_{g} &= \sqrt{\frac{1}{G}} = \sqrt{\frac{1}{0.63}} = 1.2599 \\ F_{pv} &= \sqrt{\frac{1}{Z}} = \sqrt{\frac{1}{0.876}} = 1.0684 \\ F_{aa} &= 1.001 \end{aligned} \qquad (from Table 12 on page 15) \end{aligned}$$

SO:

$$C^{I} = 1.8712 \times 10^{-5} \times 0.5856 \times 90000 \times 1 \times 0.9860 \times 1 \times 0.9459 \times 1.2599 \times 1.0684 \times 1.001 = 1.239$$
$$h_{w} = \frac{1}{P_{f}} \times \left(\frac{Q_{s}}{C^{I}}\right)^{2} = \frac{1}{8700} \times \left(\frac{1700}{1.239}\right)^{2} = 216.39 \text{ mm} \text{H}_{2}\text{O}$$

Now the value of Y_a , assumed above, can be checked:

$$Y_a = 1 - (0.011332(1 - B)^2 - 0.00342) \frac{n_w}{P_f \Upsilon}$$
 (from Equation 6 on page 8)

where:

$$B = \frac{4d}{\pi D} = \frac{4(26.924)}{300\pi} = 0.1143$$
$$H_w = 216.39 \text{mm}H_2\text{O}$$
$$P_f = 8700 \text{kPa}$$
$$\Upsilon = 1.3$$

SO:

 $Y_{a} = 1 - (0.011332(1 - 0.1143)^{2} - 0.00342)\frac{216.39}{8700 \times 1.3} = 1$

The assumed and calculated value are the same. Therefore, the value of h_w = 216.39 mm H₂O is the correct answer.